Name	B-Number

Practice Exam #2

	Your Instructor	Section #	
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☐ Jia Zhao 02	☐ Eugenia Sapir	04	☐ Thomas Zaslavsky 06

Do not open this booklet before you are told to.

Instructions — Please Read This Now

- Neatly write your **full name** and **B-Number** at the top of this page. **Check** the box next to your instructor's name. **Write your B-Number at the top of every page** — your exam will be unstapled and scanned for grading.
- You have 90 minutes to complete this exam. You may leave if you are done within the first 75 minutes. After that, we ask that you remain seated until the end of the exam.
- This exam is printed on 5 sheets of paper, double sided. There are questions on 8 of the 10 sides of the pages. Look over this exam booklet **as soon as the exam begins**. If you find any missing pages, please ask a proctor for another copy.
- Do not detach the pages of the exam booklet for any reason! If your booklet becomes unstapled for any reason, tell a proctor immediately.
- If you have any questions during the exam, raise your hand and wait for a proctor to come to your seat.
- No books, notes, cell phones, calculators, laptops, tablets, "smart watches," or any other unapproved electronic devices are allowed at any time during this exam. **Cell phones may not be taken to the restroom.**
- Unless a question states otherwise, please show all of your work and justify all of your answers. Correct answers without the appropriate work will receive zero points. Some questions have multiple parts. You do not need to duplicate the work from a previous part if it is applicable to a subsequent part.
- Please report all answers that you wish to be graded in the space provided on the exam. Please cross out anything you have written but do not want to be graded. There are three blank pages which you can use for scratch work. **You may not use your own paper.**
- You may use any of the blank pages if your work for a problem does not fit in the allotted space. You **must** indicate on the bottom of the problem page that you wish for us to consider one of the blank pages. You **must** indicate the problem number that you are completing on the blank page. Otherwise, your additional work will not be graded.
- **Academic integrity** is expected of all Binghamton University students at all times, whether in the presence or absence of members of the faculty. Understanding this, declare below that you will not give, use, or receive unauthorized aid in this examination.
- **I have read and I agree with the above instructions. I will abide by the University's Code of Academic Integrity.**

Signature of Student: _____

B-Number

This blank page is for use as scratch paper. If you use it to continue solving a problem, you must indicate the problem number above your work and you must also check the box on the bottom of problem page.

If you needed to use an extra work page for any of the problems on this page, place an X in this box. ☐

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1. (12 points) On the line provided for each part, list all possible values which answer the question. If there is no possible value, write **DNE**. No justifications are needed for this question

Suppose A is a 5×9 matrix and the associated linear transformation L_A, where $L_A(\mathbf{x}) = A\mathbf{x}$, is surjective.

Suppose B is a 7×4 matrix and the associated linear transformation L_B, where $L_B(\mathbf{x}) = B\mathbf{x}$, is injective.

(a) What are all of the possible values for the nullity of A?

(a) _____

(b) What are all of the possible values for the rank of A?

(b) _____

(c) What are all of the possible values for the rank of A^T?

(c) _____

(d) What are all of the possible values for the rank of B?

(d) _____

(e) What are all of the possible values for the nullity of B?

(e) _____

(f) What are all of the possible values for the nullity of B^T?

(f) _____

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2. (12 points) On the line for each part, provide one matrix that satisfies the condition. If no such matrix exists, write **DNE**. No justifications are needed for this question.

(a) 3×3 matrix A where $\dim(\text{Col}(A)) = 1$.

(a) _____

(b) 3×3 matrix B where $\dim(\text{Row}(B)) = 2$.

(b) _____

(c) 3×3 matrix C where $\dim(\text{Nul}(C)) = 3$.

(c) _____

(d) 2×4 matrix D where $\dim(\text{Nul}(D)) = 3$.

(d) _____

(e) 2×4 matrix E where $\dim(\text{Row}(E)) = 3$.

(e) _____

(f) 2×4 matrix F where $\dim(\text{Col}(F)) = 3$.

(f) _____

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3. (10 points) Let $S = \left\{ \begin{bmatrix} 4 & 3 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 2 & 6 \end{bmatrix} \right\}$. Extend S to a basis for $\mathbb{R}_{2\times 2}$.

4. (10 points) Let $T = \left\{ \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 4 & 4 \\ 0 & 8 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \right\}$. Reduce T to a basis for $\mathbb{R}_{2\times 2}$.

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5. (12 points) Let $\mathcal{E}$ be the standard basis for $\mathbb{R}^3$ and let $\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \right\}$ be another basis for $\mathbb{R}^3$.

(a) Find $[\mathbf{x}]_{\mathcal{E}}$, the coordinate vector of $\mathbf{x} = \begin{bmatrix} 4 \\ 7 \\ -8 \end{bmatrix}$ with respect to $\mathcal{E}$. Justify your answer.

(b) Find $[\mathbf{y}]_{\mathcal{B}}$, the coordinate vector of $\mathbf{y} = \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix}$ with respect to $\mathcal{B}$. Justify your answer.

(c) Is there a vector $\mathbf{z}$ such that $[\mathbf{z}]_{\mathcal{B}} = \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix}$? Justify your answer.

If you needed to use an extra work page for any of the problems on this page, place an X in this box. ☐

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6. (12 points) Let $L : V \to W$ be a linear transformation where $\dim(V) = 5$ and $\dim(W) = 4$.
Determine the validity of each statement as **T** (always) TRUE, **F** (always) FALSE, or **S** SOMETIMES true/false. Provide a brief justification for your answer.

(a) L is an isomorphism. (An isomorphism is a bijective linear transformation.)

(a) _____

(b) $\dim(\ker(L)) = 5$.

(b) _____

(c) If H is a subspace of V then $\dim(H) < \dim(L(H))$.

(c) _____

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7. (8 points) Let A, B, C, D be 3×3 matrices such that $\det(A) = \frac{1}{4}$, $\det(B) = \sqrt{3}$, $\det(C) = 2$, $\det(D) = 0$.

   (a) Compute $\det(AD) - \det(BC)$.

   (b) Compute $\det(B^2A^T) + \det\left(2C^{-1}\right)$.

8. (4 points) Use determinants to determine if the the three points $(-3, -8)$, $(1, -3)$, $(5, 2) \in \mathbb{R}^2$ are collinear.
   (Hint: Otherwise they would form a triangle.)

If you needed to use an extra work page for any of the problems on this page, place an X in this box. ☐

9. (10 points) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 4 & -2 & 2 \\ -7 & 1 & -6 \end{bmatrix}$.

(a) Determine if $\mathbf{e}_2 \in \text{Nul}(A)$.

(b) Determine if $\mathbf{e}_3 - \mathbf{e}_1 \in \text{Col}(A)$.

(c) Compute $\dim(\text{Row}(A)) - \dim(\text{Row}(A^T))$.

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10. (10 points) Let $A \in \mathbb{R}_{m\times p}$ and $B \in \mathbb{R}_{n\times p}$ and let $U = \{\mathbf{x} \in \mathbb{R}^p \mid A\mathbf{x} = B\mathbf{x}\}$.

(a) When $m \neq n$, U is a not a subspace of $\mathbb{R}^p$. Why not?

(b) When $m = n$, decide if U can be a subspace of $\mathbb{R}^p$. Fully justify your answer.